A QUASI-STATIC MODIFICATION OF TLM AT KNIFE EDGE AND 90° WEDGE SINGULARITIES

Lucia Cascio, Giampaolo Tardioli, Tullio Rozzi*, and Wolfgang J.R. Hoefer

NSERC/MPR Teltech Research Chair in RF Engineering. Department of Electrical and Computer Engineering, University of Victoria, Victoria, B.C. V8W3P6, CANADA

*Dipartimento di Elettronica ed Automatica, Universita' di Ancona, Ancona, I-60131, ITALY

ABSTRACT

A common drawback of numerical techniques such as TLM and FDTD resides in the difficulty to accurately describe the electromagnetic field in structures with singularities.

In this paper a local modification of the 2D-TLM algorithm for the nodes surrounding a knife edge and a 90° wedge is proposed. A quasi-static approximation of the field is used to derive an equivalent circuit of the edge.

The proposed corner correction is compared with the uncorrected TLM results and with data available in the literature, revealing a marked enhancement in the accuracy and convergence of the results.

INTRODUCTION

The TLM method [1] is widely regarded as an efficient and flexible technique for the analysis of a large class of electromagnetic problems. One of the main limitations of this and other numerical techniques is that the spatial discretization fails to accurately describe the singularities of the electromagnetic field, which occur for example close to sharp edges.

Unless a very fine discretization is used, the singular behavior around the corner is poorly represented and the frequency domain characteristics of the structure will typically be shifted. This error is very often unacceptable when we are dealing with narrowband structures such as filters.

The accuracy of the discretized model can be improved by introducing a better description of the field singularity, through local modification of the algorithm.

An approach based on the local modification of the standard TLM method to account for the energy stored around the edge has been proposed in [2]. The nodes surrounding the corner are loaded with stubs with optimized characteristics.

In this paper a new approach based on the quasi-static approximation of the Green's functions for an infinite conductive wedge is proposed. The field distribution around a corner is represented in terms of an equivalent circuit which can be implemented easily and efficiently in TLM. The accuracy of the proposed method is compared to that of the standard 2D-TLM algorithm by means of test structures for which the results are also available in the literature.

THEORY

Consider a current filament adjacent and parallel to a conducting wedge (Fig. 1) where (ρ',ϕ') indicates the source point, and (ρ,ϕ) the field point. The excitation is an impulsive current of strength I.

In this two-dimensional problem the electric field component E_z in cylindrical coordinates can be expressed as a series of trigonometric and Bessel functions [3]:

$$E_z(\rho, \phi) = G(\rho, \phi; \rho', \phi') I \qquad (1)$$

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$$G(\rho,\phi;\rho',\phi') = -\frac{\omega\mu\pi}{2(\pi-\alpha)}\sum_{\nu}\sin\nu(\phi-\alpha)\cdot$$

$$\cdot\sin\nu(\phi'-\alpha)\begin{Bmatrix} H_{\nu}^{(2)}(k\rho')J_{\nu}(k\rho) \\ H_{\nu}^{(2)}(k\rho)J_{\nu}(k\rho') \end{Bmatrix} \quad \begin{matrix}\rho<\rho' \\ \rho>\rho'\end{matrix} \tag{2}$$

$$\nu = \left.\frac{n\pi}{2(\pi-\alpha)}\right|_{n=1,2,3,\dots}$$

The expressions in (2) present a complex frequency dependance, the variable k being a function of ω. Using approximations for the Bessel and Hankel functions for small values of the argument [4], (2) can be considerably simplified leading to a quasi-static solution given by [5]:

$$G(\rho,\phi;\rho',\phi') = -j\frac{\omega\mu}{2(\pi-\alpha)}\cdot$$

$$\cdot\begin{cases} \sum_{\nu}\frac{1}{\nu}\left(\frac{\rho}{\rho'}\right)^{\nu}\sin\nu(\phi-\alpha)\sin\nu(\phi'-\alpha) & \rho<\rho' \\ \sum_{\nu}\frac{1}{\nu}\left(\frac{\rho'}{\rho}\right)^{\nu}\sin\nu(\phi-\alpha)\sin\nu(\phi'-\alpha) & \rho>\rho' \end{cases} \tag{3}$$

APPLICATION TO THE TLM MESH

The quasi-static expression for the electric field described in (3) represents the basis for the determination of an equivalent circuit describing the field around the edge.

In order to reduce the number of ports surrounding the edge, the conducting boundaries have been placed on the nodes of the TLM mesh. In the case of a knife edge ($\alpha = 0°$) a three-port equivalent circuit is required to characterize the edge behavior, while in the case of a 90° wedge ($\alpha = 45°$) a two-port equivalent circuit is sufficient (Fig. 2).

Since the voltages and currents at the ports are related to the electric field E_Z and to the current density J_Z (4), we can describe the equivalent circuit by a Z matrix representation.

$$V_i \rightarrow E_z(\rho_i,\phi_i)$$

$$I_i \rightarrow -J_z(\rho_i,\phi_i) = -\frac{I(\rho_i,\phi_i)}{2\pi\rho_i} = -\frac{I(\rho_i,\phi_i)}{\pi\Delta l} \tag{4}$$

The impedance elements depend on the Green's function as reported in (5):

$$Z_{ij} = -\pi\Delta l G(\rho_i,\phi_i;\rho'_j,\phi'_j) = \overline{G}(\rho_i,\phi_i;\rho'_j,\phi'_j) \tag{5}$$

where i and j indicate the number of the port in the circuit. The equivalent circuit is composed by inductive elements.

A more general definition for the impedances is given by:

$$Z_{ij} = \frac{1}{W_iW_j}\int_{W_i}\int_{W_j}\overline{G}(s;s_o)\,ds\,ds_o \tag{6}$$

where $\overline{G}(s;s_o)$ represents the Green's function determined in (5), and W_i, W_j are the domains of integration for the source variables and for the field variables. The adopted domains of integration are 90° circular sectors (Fig. 3).

Due to the reciprocity of the Green's function and to the geometrical symmetry of the problem there are only four distinct elements Z_{ij} for the knife edge equivalent circuit, and only two for the 90° wedge.

DISCRETIZATION PROCESS

In order to realize the equivalent circuit in the TLM mesh, we need to determine the relation between the incident and reflected voltages at the ports as a function of the Z matrix elements (6). Due to the quasi-static approximation, the voltages at the ports of the equivalent circuits depend only linearly on the frequency.

Using a bilinear discretization scheme [6] to approximate the frequency dependance, we obtain the recursive formulation (7) which describes the corner condition in the TLM process.

$$\overline{V}_k^r = \left(\frac{2}{\Delta t}Y_0[Z]-[I]\right)\left(\frac{2}{\Delta t}Y_0[Z]+[I]\right)^{-1}\cdot (\overline{V}_k^i+\overline{V}_{k-1}^r)-\overline{V}_{k-1}^i \tag{7}$$

In this expression Y_0 is the TLM link line admittance, and $\overline{V}_k^{\,r}$ and $\overline{V}_k^{\,i}$ are the vectors of the voltages incident and reflected at the terminals of the equivalent circuit at the time step k.

RESULTS

The proposed method has been applied to analyze discontinuities in the H plane of a rectangular waveguide, for both types of discontinuities.

To validate the model of a knife edge, a symmetrical inductive iris with aperture d=3.556 mm in a WR(28) waveguide has been analyzed both with the corner modification and the regular TLM algorithm, and the results have been compared with Marcuvitz's [7] formulae. The scattering parameters obtained for different discretizations are shown in Fig. 4-a.

Note that the corner modification improves considerably the accuracy of the TLM algorithm (Fig. 4-b) even when a very coarse mesh is used.

To further test the efficiency of the proposed method, an iris-coupled waveguide bandpass filter (Fig. 5), with center frequency of 33.18 GHz and bandwidth of 0.94 GHz, has been analyzed. Also in this case the corner correction results in a much faster convergence to Marcuvitz's curves as compared with the standard TLM algorithm (Fig. 6).

To verify the model the 90° wedge, a symmetrical thick iris has been examined. Comparison with the uncorrected TLM algorithm and other techniques has shown that in this case the correction is practically ineffective, since in this case the standard TLM method provides good accuracy even with coarse discretizations.

CONCLUSION

In this paper we have derived an equivalent circuit for knife edges and 90° wedges, based on a quasi-static formulation of the field around the edge, and we have introduced it in the 2D-TLM algorithm.

The proposed corner correction has been compared with the regular TLM method and with data available in the literature, and has yielded a noticeable improvement in the accuracy as well as in the convergence of the results for knife edges, while in the case of 90° wedges the standard TLM algorithm has proved to be sufficiently accurate.

The better description of the singular behavior of the field around the edge allows considerable savings in computer processing time and memory requirements when compared to mesh grading, since the desired accuracy can be achieved by using a coarser lattice.

An immediate extension of this method is its application to problems involving dielectric interfaces and sharp metallic boundaries, such as microstrips.

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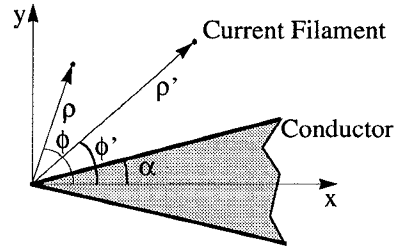

Fig. 1 Conducting wedge

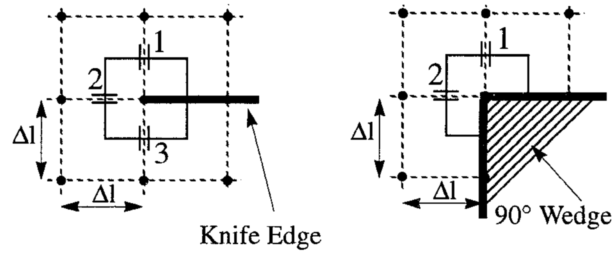

Fig. 2 Knife edge and 90° wedge position in the TLM mesh

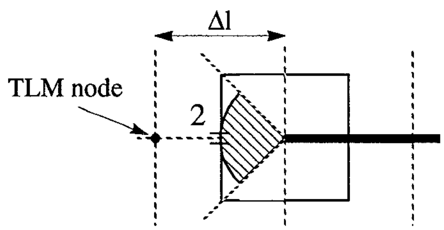

Fig. 3 Domain of integration for the determination of Z_{ij}

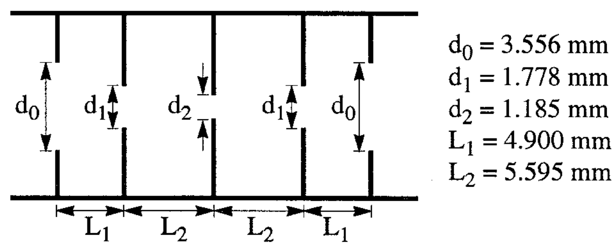

Fig. 5 Top view of the iris coupled bandpass filter in WR(28) waveguide

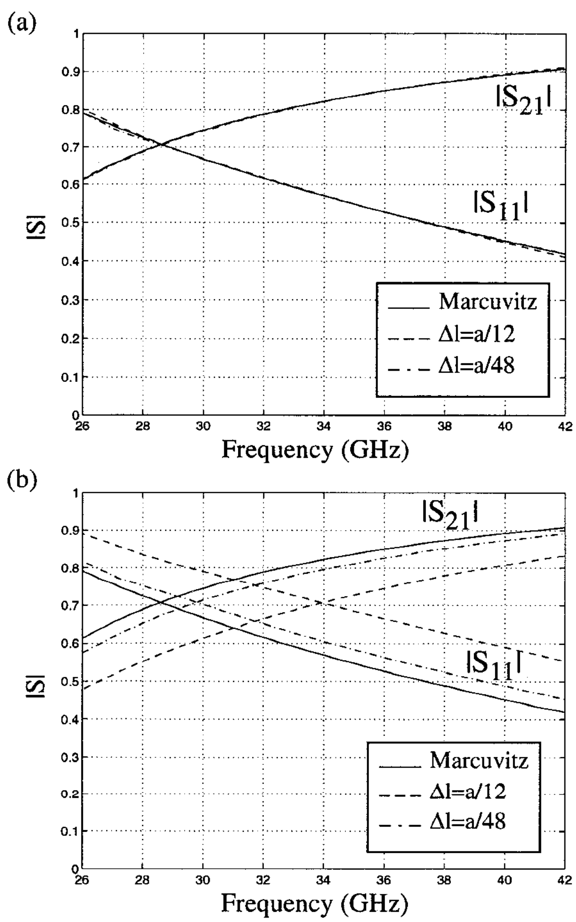

Fig. 4 S-parameters for an inductive iris in WR(28) waveguide: a) TLM with corner correction, b) TLM without corner correction

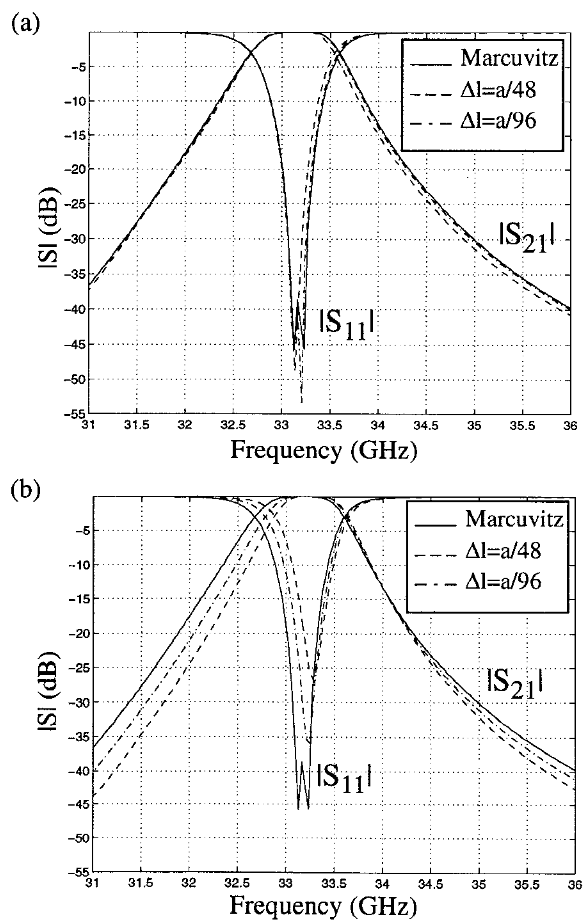

Fig. 6 Iris coupled bandpass filter in WR(28) waveguide: a) TLM with corner correction, b) TLM without corner correction